Department of Artificial Intelligence

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Empowering Small-Scale Knowledge Graphs: A Strategy of Leveraging General-Purpose Knowledge Graphs for Enriched Embeddings

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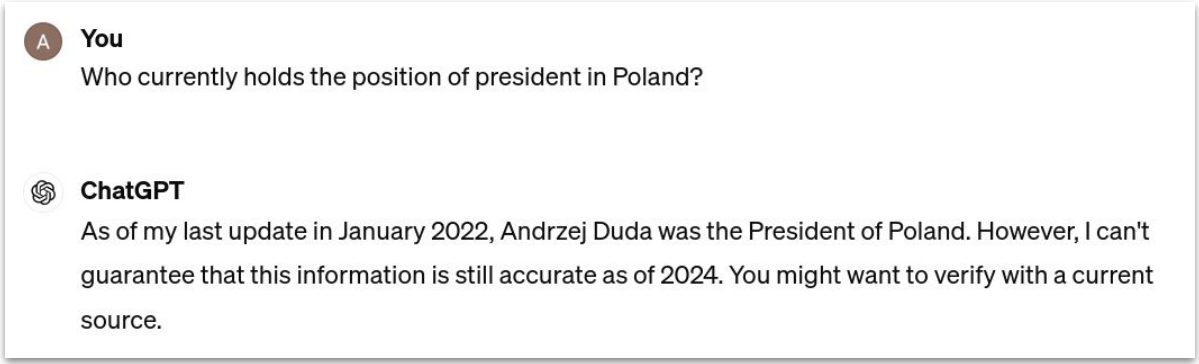
Plan of the presentation

1. Introduction
2. Motivation
3. Framework
4. Experimental methodology
5. Results
6. Conclusion & future work

Introduction

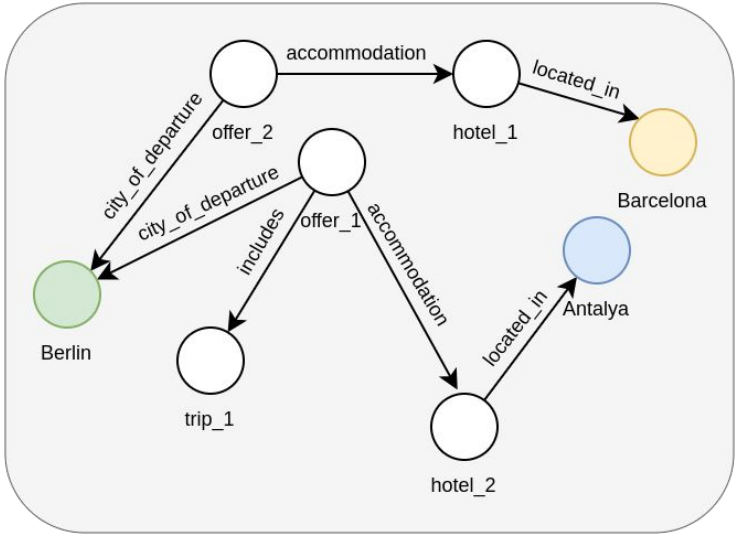
- Availability of advanced AI methods has contributed to the growth of applications in knowledge-intensive tasks, e.g.:
  - QA
  - Fact-checking
- Limitations of LLM in updating facts after training: outdated or incorrect information (hallucinations).



Conversation with ChatGPT (26.04.2024)

Motivation

- Leveraging Knowledge Graphs (KGs) holds promise in addressing challenges of knowledge-intensive applications.
- Development of domain-specific knowledge graphs is hindered by cost and complexity.
- Small KGs suffer from:
  - limited relational structures
  - sparse entity interactions
  - reduced contextual information

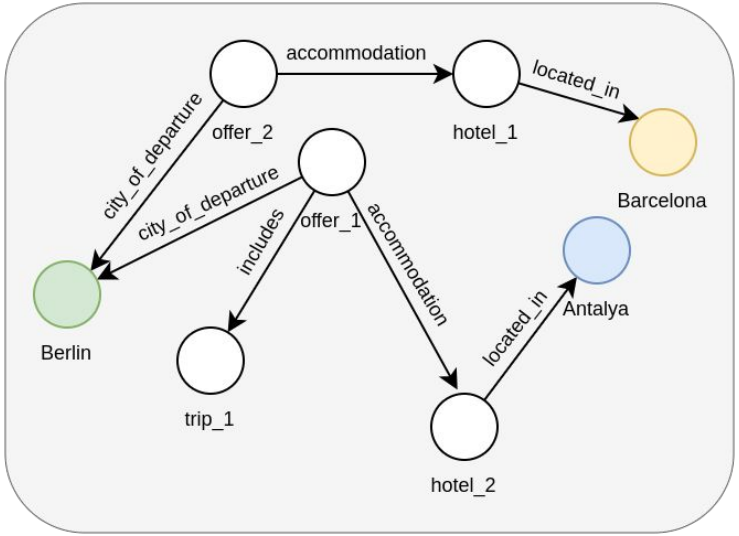


Example of KG.

Motivation

- Empowering small-scale KGs will facilitate the development of KG-based systems, particularly for:
  - Small companies and startups
  - Research groups and academia
  - Less popular or niche domains

- How can we utilize existing resources for that?



Example of KG.

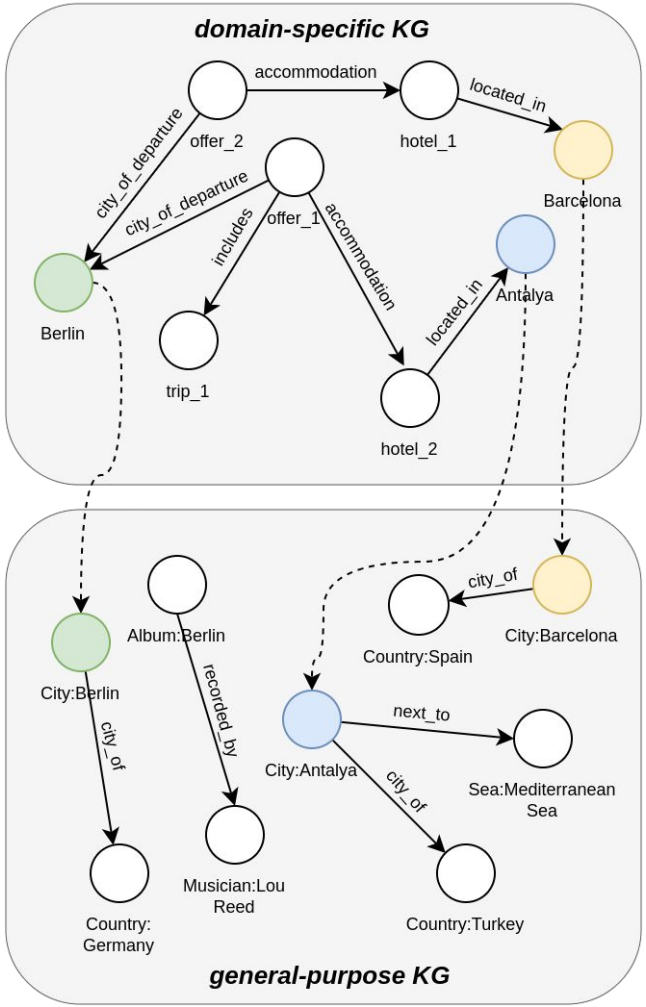
Motivation

- **Idea:**

  Integrating external KG as knowledge source to empower small-scale KG.

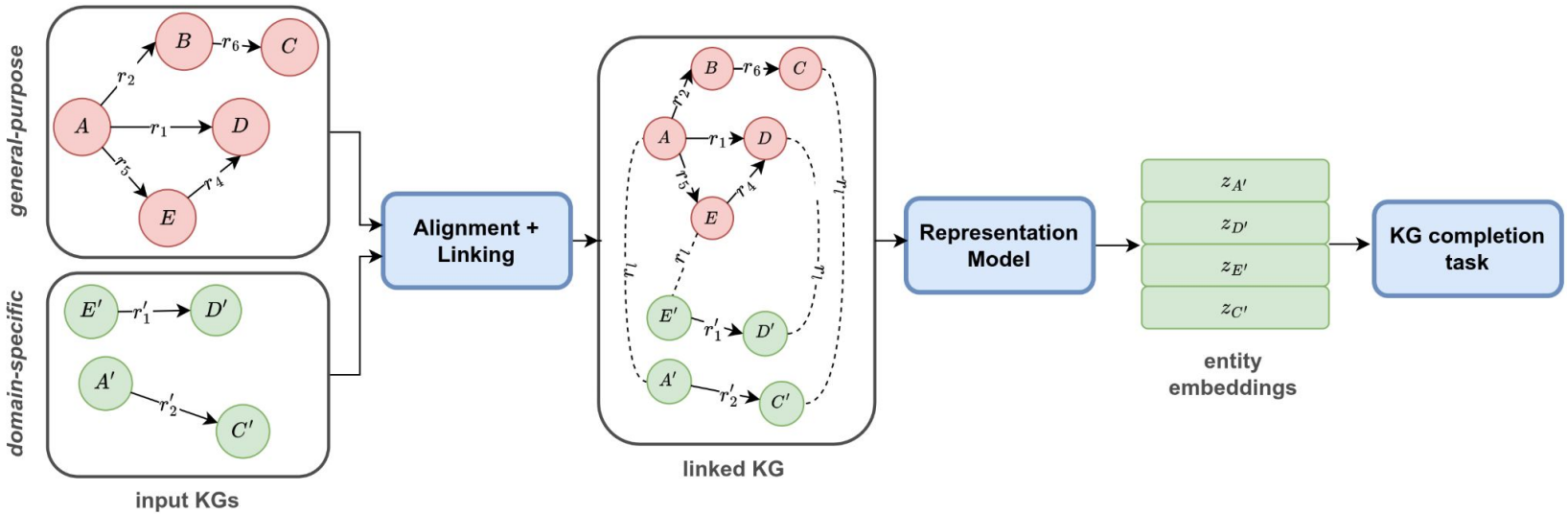
- **Aim:**

  Developing a framework for enriching vector representations of domain-specific knowledge graphs by linking them with well-established, general-purpose KGs.



Framework

- We propose a generic and modular framework for enriching small and domain-specific KGs (DKG) with general-purpose KGs (GKG) using alignment and linking operation:

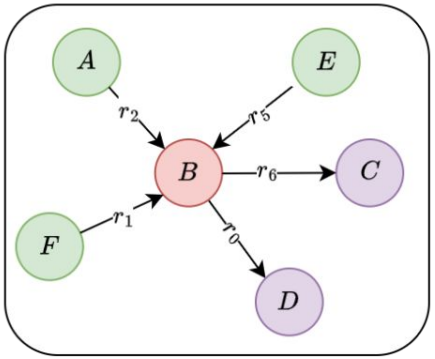


Alignment and linking

We perform alignment and linking without requiring manual annotation, leveraging the textual entity labels:

1. We acquire a representation vector $x(e_i)$ for each entity e_i based on three components:

$$x(e_i) = \left[\phi(e_i) \,\middle\|\, \sum_{e_u \in \mathcal{O}(e_i)} \frac{\phi(e_u)}{|\mathcal{O}(e_i)|} \,\middle\|\, \sum_{e_v \in \mathcal{I}(e_i)} \frac{\phi(e_v)}{|\mathcal{I}(e_i)|} \right]$$



$$x(B) = \left[\phi(B) \,\middle\|\, \frac{\phi(C) + \phi(D)}{2} \,\middle\|\, \frac{\phi(A) + \phi(E) + \phi(F)}{3} \right]$$

2. For each entity in the DKG, we find *k* nearest neighbors in the GKG, based on the vectors $x(\cdot)$.
3. For each pair of neighbors found, we create an artificial triple $(e_i^{(d)}, r_l, e_j^{(g)})$.

Representation learning

- For the linked graph $\mathcal{G}_l = (\mathcal{E}_d \cup \mathcal{E}_g, \mathcal{R}_d \cup \mathcal{R}_g \cup \{r_l\}, \mathcal{T}_d \cup \mathcal{T}_g \cup \mathcal{T}_l)$, we train the representation model and evaluate its performance on KG completion task.
- We propose a weighted loss function to mitigate entity alignment's negative effects between two considered KGs:

$$\mathcal{L}(\mathcal{T}, \tilde{\mathcal{T}}; \theta) = \sum_{t \in \mathcal{T}_d \cup \mathcal{T}_g} L(t, \tilde{\mathcal{T}}; \theta) + \sum_{s \in \mathcal{T}_l} w_s \cdot L(s, \tilde{\mathcal{T}}; \theta)$$

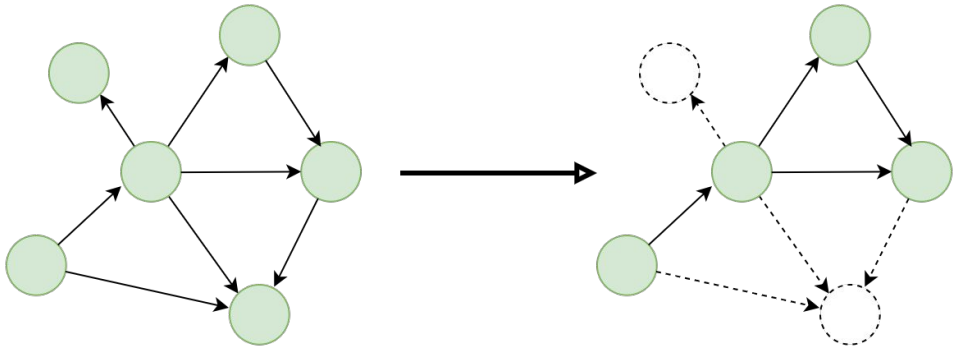
$$w_s = \frac{1}{1 + distance(x(e_i), x(e_j))}$$

$\mathcal{T}$: set of positive triples
$\tilde{\mathcal{T}}$: set of sampled negative triples
$\mathcal{T}_d$: triples from DKG
$\mathcal{T}_g$: triples from GKG
$\mathcal{T}_l$: set of linking triples

Experimental methodology

- Existing datasets are not suitable for evaluating methods aimed at enriching the embeddings of KGs.
- We designed a custom evaluation procedures tailored to this context.
- We simulated different stages in developing a KG by:
  1. **Triple sampling**
  2. **Node sampling**
  3. **Relation sampling**

  Each strategy is parameterized by the probability p of keeping a triple, node, or relation in the sampled graph.



Experimental methodology

We conducted empirical studies in synthetic and real-world scenarios:

- **Synthetic scenario:**

  - controlled environment
  - perfect overlap of knowledge domains
  - simplified evaluation

- **Real-world scenario:**

  - external KG differs from DKG
  - mimics real scenarios
  - limited experimental control

Results: Synthetic scenario

Experiments settings:

- **Datasets:** WN18RR, FB15k-237, WD50K
- **Sampling:** triple, node, relation
- ***p:*** 0.4, 0.6, 0.8
- **Representation model:** *RotatE*

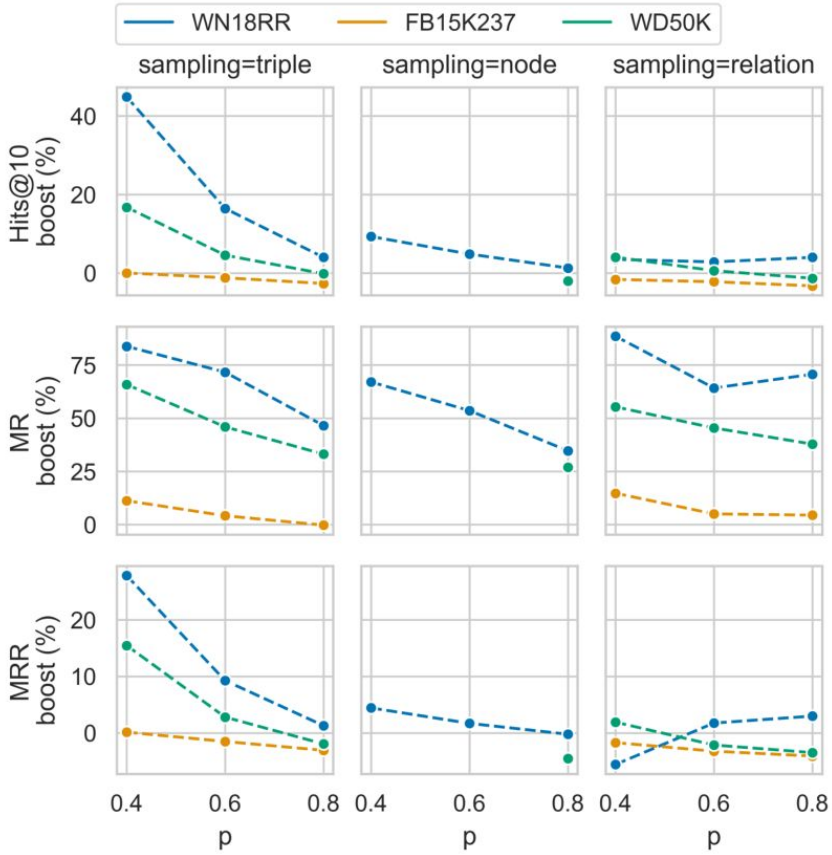
Results on WN18RR (more results in the paper).

sampling	p	$Hits@10 \uparrow$ single	$Hits@10 \uparrow$ linked	boost(%)	$MR \downarrow$ single	$MR \downarrow$ linked	boost(%)	$MRR \uparrow$ single	$MRR \uparrow$ linked	boost(%)
					WN18RR					
triple	0.4	0.347 ± 0.005	$\mathbf{0.502 \pm 0.006}$	44.9	7681 ± 62	$\mathbf{1245 \pm 44}$	83.8	0.270 ± 0.003	$\mathbf{0.345 \pm 0.001}$	27.8
triple	0.6	0.446 ± 0.001	$\mathbf{0.519 \pm 0.004}$	16.4	4908 ± 172	$\mathbf{1392 \pm 15}$	71.6	0.342 ± 0.003	$\mathbf{0.373 \pm 0.004}$	9.3
triple	0.8	0.525 ± 0.004	$\mathbf{0.546 \pm 0.004}$	4.0	2685 ± 155	$\mathbf{1435 \pm 52}$	46.6	0.416 ± 0.005	$\mathbf{0.421 \pm 0.003}$	1.3
node	0.4	0.546 ± 0.004	$\mathbf{0.597 \pm 0.002}$	9.3	2164 ± 52	$\mathbf{713 \pm 18}$	67.0	0.473 ± 0.002	$\mathbf{0.494 \pm 0.001}$	4.4
node	0.6	0.562 ± 0.000	$\mathbf{0.590 \pm 0.001}$	4.8	2044 ± 29	$\mathbf{950 \pm 15}$	53.6	0.480 ± 0.001	$\mathbf{0.488 \pm 0.001}$	1.7
node	0.8	0.576 ± 0.004	$\mathbf{0.583 \pm 0.001}$	1.3	1829 ± 8	$\mathbf{1194 \pm 20}$	34.7	$\mathbf{0.484 \pm 0.001}$	0.482 ± 0.001	-0.2
relation	0.4	0.696 ± 0.182	$\mathbf{0.721 \pm 0.184}$	3.5	1083 ± 450	$\mathbf{124 \pm 70}$	88.5	$\mathbf{0.590 \pm 0.233}$	0.557 ± 0.212	-5.6
relation	0.6	0.755 ± 0.129	$\mathbf{0.777 \pm 0.143}$	2.8	1542 ± 622	$\mathbf{552 \pm 649}$	64.2	0.684 ± 0.145	$\mathbf{0.696 \pm 0.160}$	1.7
relation	0.8	0.731 ± 0.115	$\mathbf{0.760 \pm 0.135}$	4.0	2013 ± 157	$\mathbf{591 \pm 671}$	70.7	0.659 ± 0.130	$\mathbf{0.678 \pm 0.149}$	3.0
Max boost (%)				44.9			88.5			27.8
Mean boost (%)				10.1			64.5			4.8

Results: Synthetic scenario

Key Findings:

- Significant improvement by linking GKG.
- Effectiveness depends on sampling setting.
- Effectiveness varies among datasets due to inherent characteristics.
- **Achieved *44.9% / 0.0% / 16.7% Hits@10* boost on WN18RR / FB15k-237 / WD50K with only 40% of triples.**



Results: Real-world scenario

Experiments settings:

- **Datasets pairs:**
  - WN18RR - ConceptNet
  - WN18RR - FB15k-237
  - FB15k-237 - ConceptNet
  - FB15k-237 - YAGO3-10
  - WD50K - FB15k-237
  - WD50K - YAGO3-10
- **Sampling:** triple
- ***p:*** 0.4, 0.6, 0.8
- **Representation model:** *RotatE*

Results on WN18RR (more results in the paper).

dataset	p	$Hits@10 \uparrow$ single	$Hits@10 \uparrow$ linked CN	$Hits@10 \uparrow$ linked FB	$Hits@10 \uparrow$ boost(%)	$MR \downarrow$ single	$MR \downarrow$ linked CN	$MR \downarrow$ linked FB	$MR \downarrow$ boost(%)	$MRR \uparrow$ single	$MRR \uparrow$ linked CN	$MRR \uparrow$ linked FB	$MRR \uparrow$ boost(%)
WN18RR	0.4	0.347 ± 0.005	**0.395**	0.338	14.0	7681 ± 62	**1508**	4050	80.4	$\mathbf{0.270 \pm 0.003}$	0.259	0.262	−3.2
WN18RR	0.6	0.446 ± 0.001	**0.471**	0.435	5.7	4908 ± 172	**963**	2685	80.4	$\mathbf{0.342 \pm 0.003}$	0.330	0.330	−3.3
WN18RR	0.8	0.525 ± 0.004	**0.527**	0.504	0.3	2685 ± 155	**750**	1725	72.1	$\mathbf{0.416 \pm 0.005}$	0.390	0.400	−3.8
Max boost (%)					14.0				80.4				−3.2
Mean boost (%)					6.7				77.6				−3.5

Results: Real-world scenario

Key Findings:

- Highest efficacy improvement observed under rigorous conditions (40% of triples).
- Achieved notable Hits@10 boost on WN18RR-ConceptNet and WD50K-FB15k-237 pairs, suggesting framework effectiveness.
    - WN18RR – ConceptNet:
    - WD50K – FB15k-237:
- Importance of choosing a well-suited GKG for the DKG.

Conclusion

Framework

- Proposed general and modular framework for enriching embeddings of small scale KGs.
- Utilized straightforward alignment method based on textual embeddings of entities and their neighborhood.
- Proposed weighted loss function mitigates negative effects of entity alignment.

Extensive Experimentation

- Conducted evaluation in synthetic and real-world scenarios, that simulate early KG development stages.
- Results show significant performance improvement on downstream tasks.
- Degree of improvement varies based on specific DKG and linked general-purpose GKG.

Implications and Future Directions

- Utilizing GKGs strengthens emerging KGs, enhancing their utility and effectiveness.
- Research signals potential pathway for future exploration in enriching small-scale KGs.

Acknowledgements

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- Horizon Europe Framework Programme MSCA Staff Exchanges grant no. 101086321 (OMINO)
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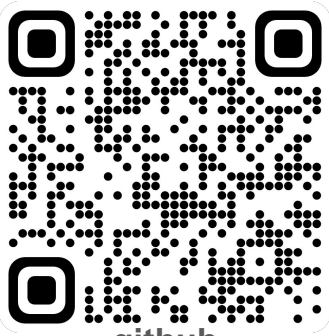
- Department of Artificial Intelligence, Wroclaw Tech

Thank you for your attention!

- **Code:** *github.com/graphml-lab-pwr/empowering-small-scale-kg*
- **Contact:** albert.sawczyn@pwr.edu.pl



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Appendix

Results: Synthetic scenario

sampling	p	$Hits@10 \uparrow$ single	$Hits@10 \uparrow$ linked	$Hits@10 \uparrow$ boost(%)	$MR \downarrow$ single	$MR \downarrow$ linked	$MR \downarrow$ boost(%)	$MRR \uparrow$ single	$MRR \uparrow$ linked	$MRR \uparrow$ boost(%)
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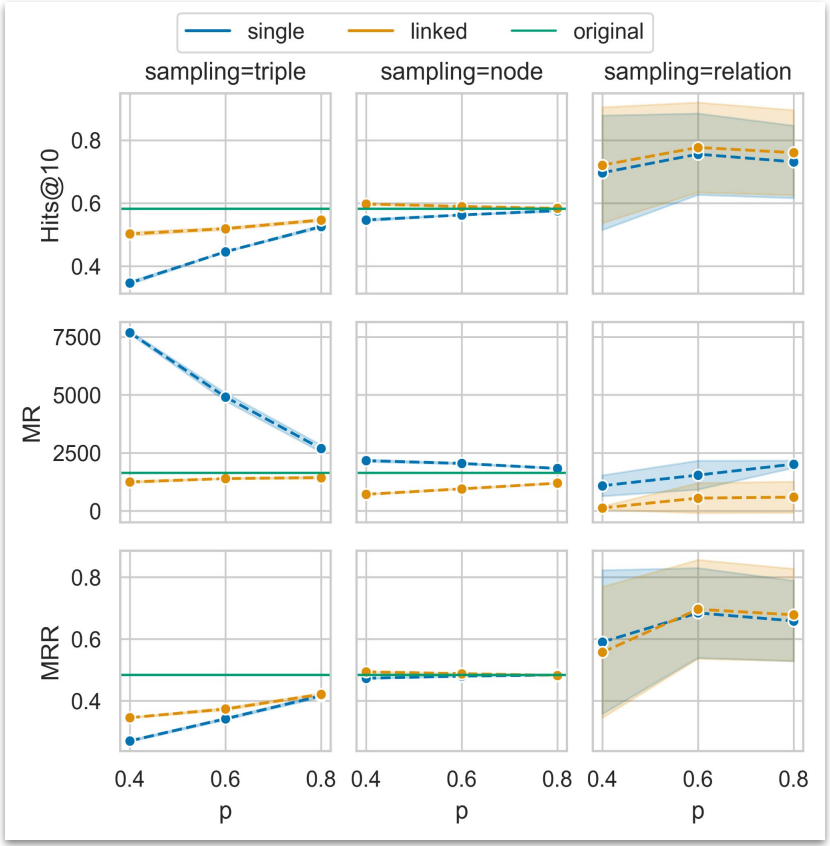
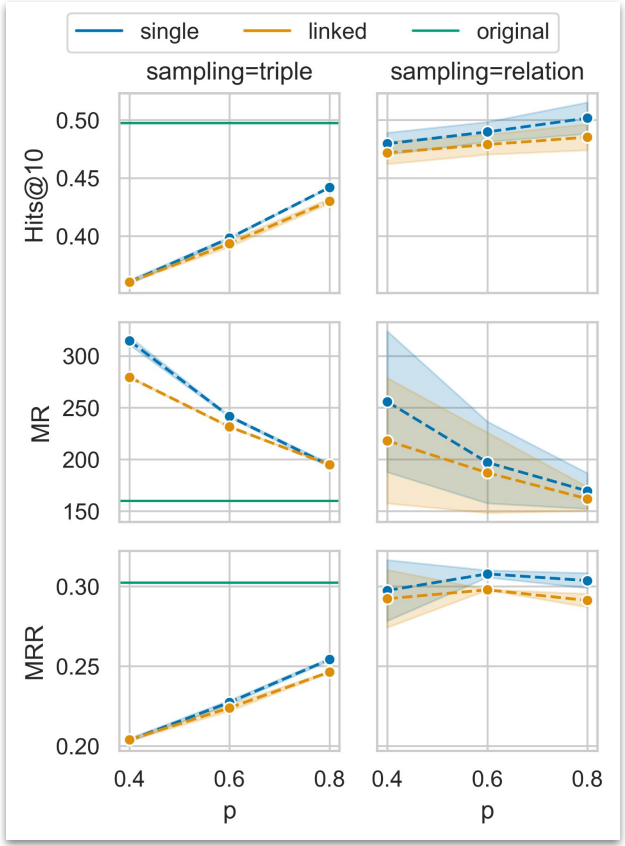
Max boost (%)				44.9			88.5			27.8
Mean boost (%)				10.1			64.5			4.8
FB15k-237										
triple	0.4	$\mathbf{0.360 \pm 0.002}$	0.360 ± 0.000	0.0	315 ± 4	$\mathbf{279 \pm 1}$	11.2	0.204 ± 0.001	$\mathbf{0.204 \pm 0.001}$	0.1
triple	0.6	$\mathbf{0.398 \pm 0.001}$	0.393 ± 0.003	−1.2	242 ± 1	$\mathbf{231 \pm 0}$	4.2	$\mathbf{0.227 \pm 0.001}$	0.224 ± 0.002	−1.5
triple	0.8	$\mathbf{0.442 \pm 0.000}$	0.430 ± 0.002	−2.7	$\mathbf{194 \pm 2}$	195 ± 1	−0.3	$\mathbf{0.254 \pm 0.001}$	0.246 ± 0.000	−3.1
relation	0.4	$\mathbf{0.480 \pm 0.009}$	0.472 ± 0.010	−1.6	256 ± 68	$\mathbf{218 \pm 61}$	14.7	$\mathbf{0.297 \pm 0.019}$	0.292 ± 0.018	−1.7
relation	0.6	$\mathbf{0.490 \pm 0.009}$	0.479 ± 0.009	−2.2	197 ± 39	$\mathbf{187 \pm 39}$	5.0	$\mathbf{0.308 \pm 0.002}$	0.298 ± 0.000	−3.2
relation	0.8	$\mathbf{0.502 \pm 0.014}$	0.485 ± 0.011	−3.3	169 ± 17	$\mathbf{162 \pm 12}$	4.4	$\mathbf{0.304 \pm 0.005}$	0.291 ± 0.004	−4.1
Max boost (%)				0.0			14.7			0.1
Mean boost (%)				−1.8			6.5			−2.3
WD50K										
triple	0.4	0.283 ± 0.001	$\mathbf{0.330 \pm 0.000}$	16.7	1713 ± 42	$\mathbf{585 \pm 3}$	65.8	0.164 ± 0.001	$\mathbf{0.189 \pm 0.001}$	15.5
triple	0.6	0.353 ± 0.000	$\mathbf{0.369 \pm 0.001}$	4.5	931 ± 20	$\mathbf{503 \pm 5}$	45.9	0.208 ± 0.001	$\mathbf{0.213 \pm 0.000}$	2.8
triple	0.8	$\mathbf{0.400 \pm 0.001}$	0.399 ± 0.001	−0.2	667 ± 6	$\mathbf{446 \pm 4}$	33.1	$\mathbf{0.240 \pm 0.001}$	0.235 ± 0.000	−2.0
node	0.8	$\mathbf{0.440 \pm 0.002}$	0.431 ± 0.001	−2.0	520 ± 2	$\mathbf{380 \pm 1}$	27.0	$\mathbf{0.271 \pm 0.001}$	0.259 ± 0.002	−4.6
relation	0.4	0.413 ± 0.023	$\mathbf{0.430 \pm 0.029}$	4.0	768 ± 123	$\mathbf{343 \pm 97}$	55.3	0.249 ± 0.025	$\mathbf{0.254 \pm 0.027}$	1.9
relation	0.6	0.437 ± 0.014	$\mathbf{0.439 \pm 0.007}$	0.6	617 ± 42	$\mathbf{337 \pm 64}$	45.4	$\mathbf{0.269 \pm 0.013}$	0.263 ± 0.008	−2.2
relation	0.8	$\mathbf{0.447 \pm 0.016}$	0.441 ± 0.008	−1.3	541 ± 83	$\mathbf{336 \pm 67}$	37.8	$\mathbf{0.275 \pm 0.010}$	0.266 ± 0.005	−3.5
Max boost (%)				16.7			65.8			15.5
Mean boost (%)				3.2			44.4			1.1

Results: Synthetic scenario

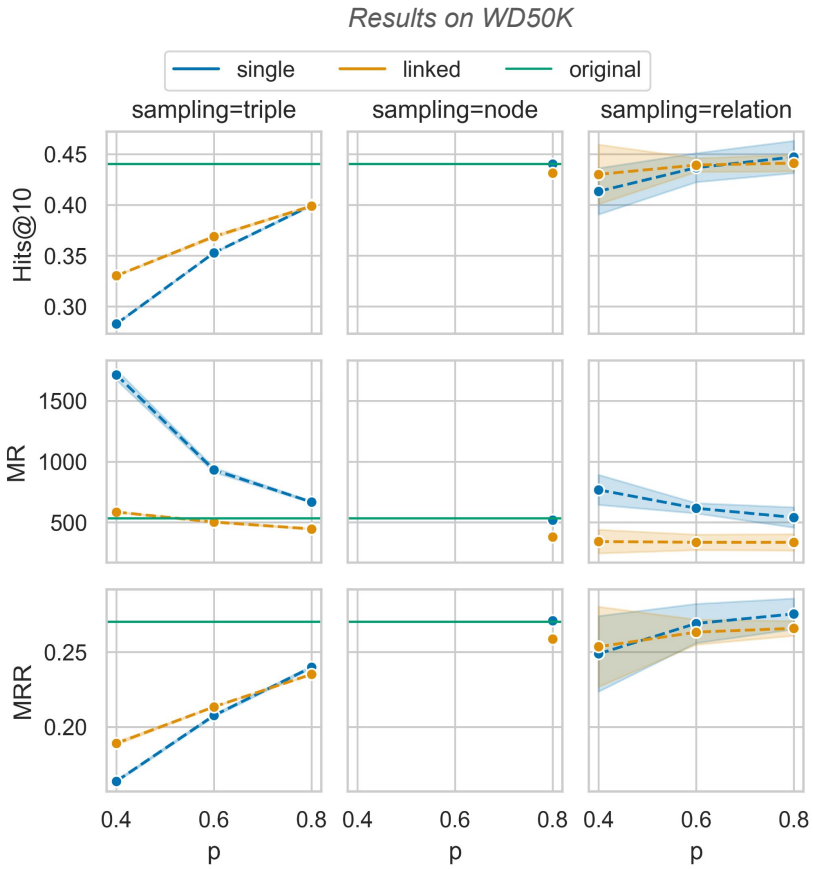
Results on WN18RR



Results on FB15k-237



Results: Synthetic scenario

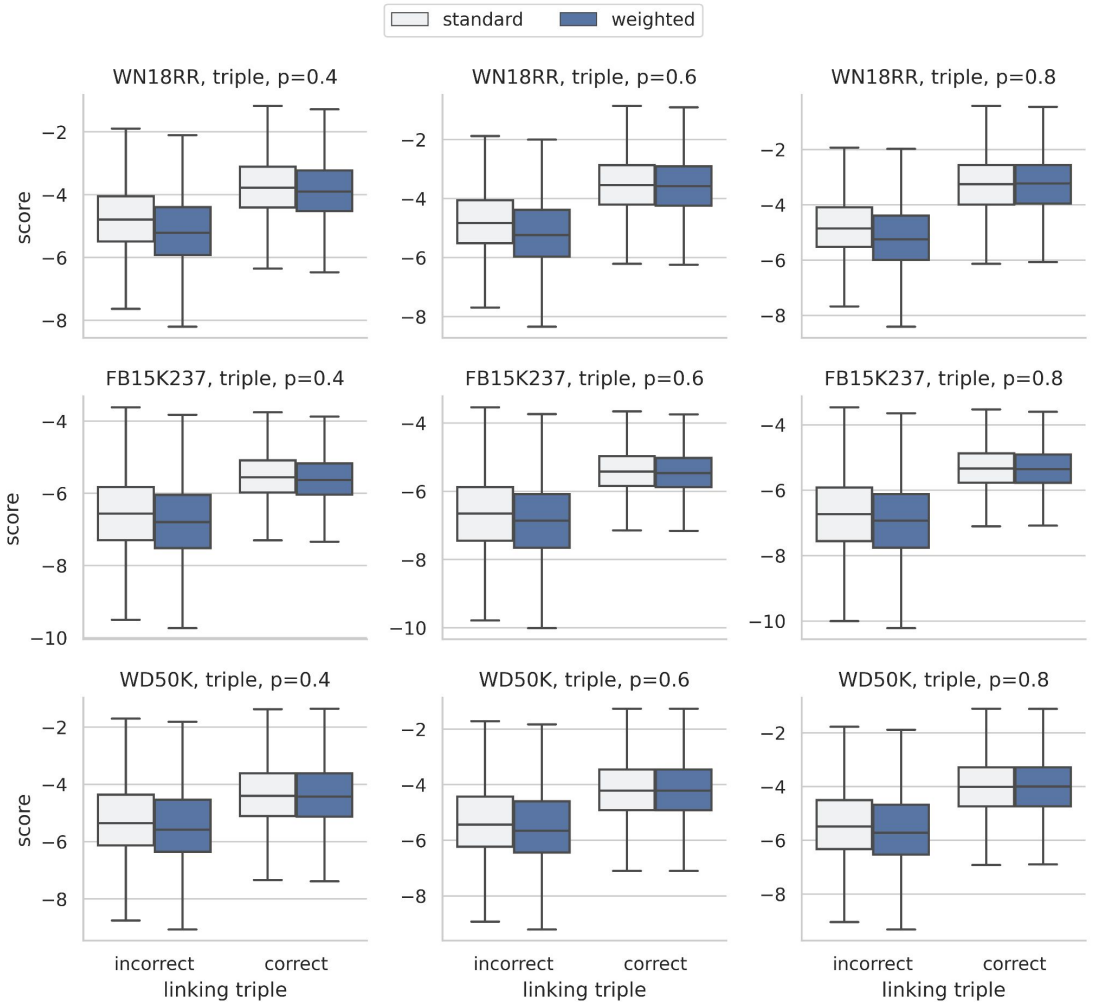


Results: Real-world scenario

dataset	p	$Hits@10 \uparrow$ single	$Hits@10 \uparrow$ linked		$Hits@10 \uparrow$ boost(%)	$MR \downarrow$ single	$MR \downarrow$ linked		$MR \downarrow$ boost(%)	$MRR \uparrow$ single	$MRR \uparrow$ linked		$MRR \uparrow$ boost(%)
			CN	FB			CN	FB			CN	FB	
WN18RR	0.4	0.347 ± 0.005	**0.395**	0.338	14.0	7681 ± 62	**1508**	4050	80.4	$\mathbf{0.270 \pm 0.003}$	0.259	0.262	−3.2
WN18RR	0.6	0.446 ± 0.001	**0.471**	0.435	5.7	4908 ± 172	**963**	2685	80.4	$\mathbf{0.342 \pm 0.003}$	0.330	0.330	−3.3
WN18RR	0.8	0.525 ± 0.004	**0.527**	0.504	0.3	2685 ± 155	**750**	1725	72.1	$\mathbf{0.416 \pm 0.005}$	0.390	0.400	−3.8
Max boost (%)					14.0				80.4				−3.2
Mean boost (%)					6.7				77.6				−3.5
			CN	Y3-10			CN	Y3-10			CN	Y3-10	
FB15K237	0.4	$\mathbf{0.360 \pm 0.002}$	0.330	0.335	−6.9	315 ± 4	315	**310**	1.4	$\mathbf{0.204 \pm 0.001}$	0.189	0.192	−5.6
FB15K237	0.6	$\mathbf{0.398 \pm 0.001}$	0.359	0.362	−9.1	$\mathbf{242 \pm 1}$	254	251	−3.9	$\mathbf{0.227 \pm 0.001}$	0.202	0.208	−8.6
FB15K237	0.8	$\mathbf{0.442 \pm 0.000}$	0.387	0.395	−10.6	$\mathbf{194 \pm 2}$	213	214	−9.5	$\mathbf{0.254 \pm 0.001}$	0.221	0.225	−11.5
Max boost (%)					−6.9				1.4				−5.6
Mean boost (%)					−8.9				−4.0				−8.6
			FB	Y3-10			FB	Y3-10			FB	Y3-10	
WD50K	0.4	0.283 ± 0.001	**0.313**	0.284	10.8	1713 ± 42	**758**	961	55.8	0.164 ± 0.001	**0.181**	0.163	10.6
WD50K	0.6	0.353 ± 0.000	**0.361**	0.327	2.4	931 ± 20	**558**	644	40.0	0.208 ± 0.001	**0.212**	0.190	2.0
WD50K	0.8	$\mathbf{0.400 \pm 0.001}$	0.396	0.358	−0.9	667 ± 6	**450**	510	32.6	$\mathbf{0.240 \pm 0.001}$	0.236	0.210	−1.8
Max boost (%)					10.8				55.8				10.6
Mean boost (%)					4.1				42.8				3.6

Ablation study: loss

Comparative analysis of predicted scores on the training set



Statistics of the original datasets

	Triples			Entities			Relations		
dataset	train	val	test	train	val	test	train	val	test
WN18RR*	86 835	2817	2923	40 714	4835	4985	11	11	11
FB15k-237*†	272 115	17 526	20 438	14 505	9799	10 317	237	237	237
WD50K*	164 631	22 429	45 284	40 107	16 500	22 732	473	297	347
ConceptNet†	3 423 004	0	0	1 787 373	0	0	47	47	47
YAGO3-10†	1 079 040	4978	4982	123 143	7914	7906	37	37	37

Statistics of the sampled datasets

sampling	p	Triples			Entities			Relations		
		train	val	test	train	val	test	train	val	test
WN18RR										
triple	0.4	34 734 ± 0	2817 ± 0	2923 ± 0	30 975 ± 57	4835 ± 0	4985 ± 0	11 ± 0	11 ± 0	11 ± 0
triple	0.6	52 101 ± 0	2817 ± 0	2923 ± 0	36 178 ± 27	4835 ± 0	4985 ± 0	11 ± 0	11 ± 0	11 ± 0
triple	0.8	69 468 ± 0	2817 ± 0	2923 ± 0	39 093 ± 28	4835 ± 0	4985 ± 0	11 ± 0	11 ± 0	11 ± 0
node	0.4	28 815 ± 78	2817 ± 0	2923 ± 0	16 286 ± 0	4835 ± 0	4985 ± 0	11 ± 0	11 ± 0	11 ± 0
node	0.6	46 715 ± 90	2817 ± 0	2923 ± 0	24 428 ± 0	4835 ± 0	4985 ± 0	11 ± 0	11 ± 0	11 ± 0
node	0.8	66 311 ± 157	2817 ± 0	2923 ± 0	32 571 ± 0	4835 ± 0	4985 ± 0	11 ± 0	11 ± 0	11 ± 0
relation	0.4	20 367 ± 18 482	511 ± 625	545 ± 616	14 485 ± 10 417	933 ± 1168	989 ± 1150	4 ± 0	4 ± 0	4 ± 0
relation	0.6	57 790 ± 19 777	1765 ± 624	1827 ± 635	32 006 ± 7832	3135 ± 1000	3228 ± 1025	7 ± 0	7 ± 0	7 ± 0
relation	0.8	61 586 ± 17 153	1852 ± 598	1902 ± 610	34 809 ± 4855	3286 ± 949	3353 ± 974	9 ± 0	9 ± 0	9 ± 0
FB15k-237										
triple	0.4	108 846 ± 0	17 526 ± 0	20 438 ± 0	14 187 ± 10	9799 ± 0	10 317 ± 0	237 ± 0	223 ± 0	224 ± 0
triple	0.6	163 269 ± 0	17 526 ± 0	20 438 ± 0	14 352 ± 6	9799 ± 0	10 317 ± 0	237 ± 0	223 ± 0	224 ± 0
triple	0.8	217 692 ± 0	17 526 ± 0	20 438 ± 0	14 439 ± 7	9799 ± 0	10 317 ± 0	237 ± 0	223 ± 0	224 ± 0
relation	0.4	103 999 ± 16 860	6364 ± 891	7419 ± 1056	12 884 ± 220	5457 ± 707	5981 ± 722	95 ± 0	87 ± 2	89 ± 3
relation	0.6	156 537 ± 20 322	9562 ± 1011	11 186 ± 1268	13 779 ± 352	7218 ± 571	7747 ± 589	142 ± 0	133 ± 3	134 ± 3
relation	0.8	224 750 ± 25 236	14 309 ± 1632	16 748 ± 1881	14 332 ± 75	8725 ± 633	9291 ± 618	190 ± 0	178 ± 2	180 ± 2
WD50K										
triple	0.4	65 852 ± 0	22 429 ± 0	45 284 ± 0	34 164 ± 32	16 500 ± 0	22 732 ± 0	434 ± 2	297 ± 0	347 ± 0
triple	0.6	98 779 ± 0	22 429 ± 0	45 284 ± 0	37 091 ± 41	16 500 ± 0	22 732 ± 0	453 ± 6	297 ± 0	347 ± 0
triple	0.8	131 705 ± 0	22 429 ± 0	45 284 ± 0	38 957 ± 16	16 500 ± 0	22 732 ± 0	464 ± 3	297 ± 0	347 ± 0
node	0.8	147 774 ± 42	22 429 ± 0	45 284 ± 0	32 086 ± 0	16 500 ± 0	22 732 ± 0	450 ± 1	297 ± 0	347 ± 0
relation	0.4	62 701 ± 11 210	7865 ± 1591	16 014 ± 3259	25 204 ± 2662	7700 ± 956	11 535 ± 1186	189 ± 0	112 ± 6	127 ± 5
relation	0.6	88 487 ± 12 893	11 529 ± 1838	23 332 ± 3923	29 090 ± 3143	10 258 ± 1183	14 848 ± 1580	284 ± 0	173 ± 5	201 ± 6
relation	0.8	121 499 ± 15 858	16 250 ± 2317	32 947 ± 4969	33 181 ± 2970	12 848 ± 1161	18 053 ± 1489	378 ± 0	238 ± 4	275 ± 5