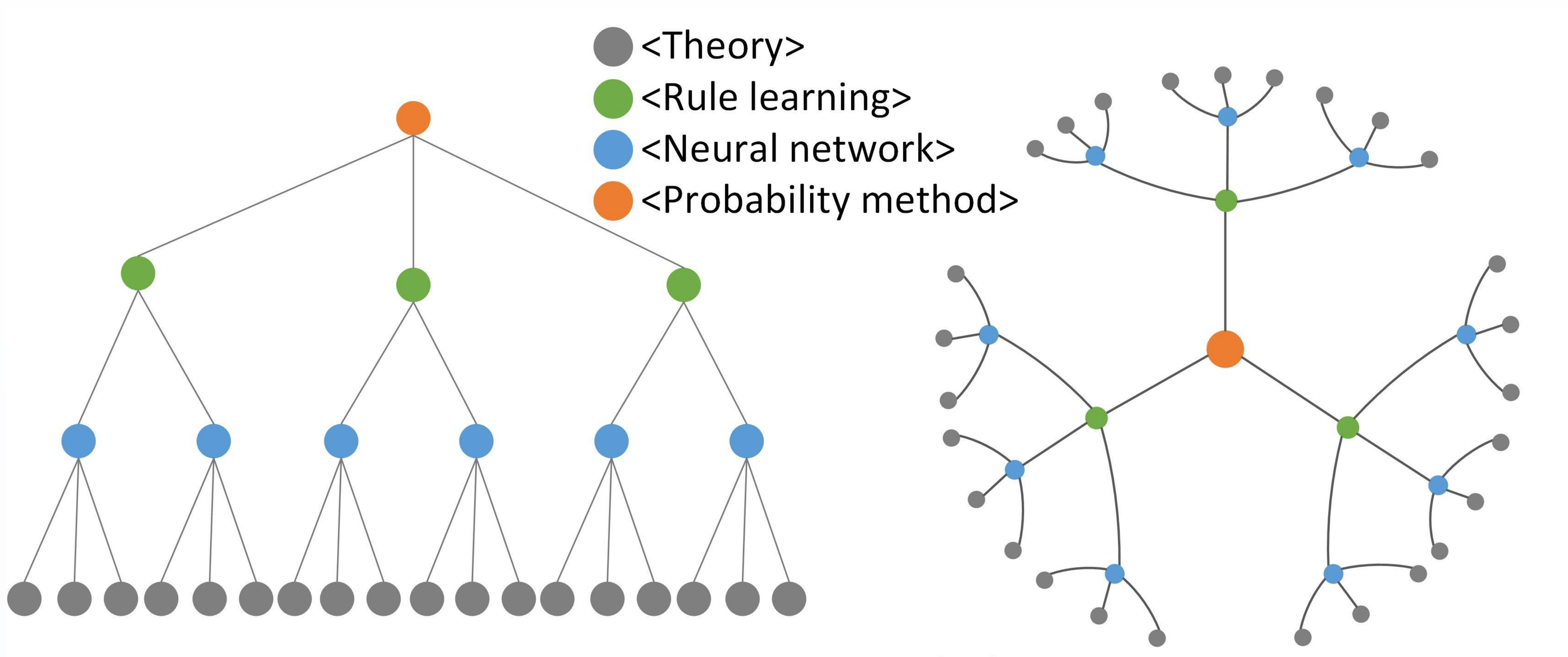
L²GC: Lorentzian Linear Graph Convolutional Networks For Node Classification

Qiuyu Liang, Weihua Wang *, Feilong Bao, Guanglai Gao

College of Computer Science, Inner Mongolia University

Motivation

- Linear Graph Convolutional Networks are used to classify nodes in graph data.
- Existing linear GCN models don't capture the tree-like structure of real-world datasets.
- Hyperbolic spaces have a greater capacity to represent scale-free or hierarchical structures, which can be used to learn tree-like graphs.



(a) Euclidean space (b) Hyperbolic space

Our Method

- Parameter-Free propagation. Get the graph node features from ***n***-power propagations.

$$H^{(l+1)} = (1-\alpha)PH^{(l)} + \alpha X,$$

- Mapping the learned features of graph nodes into hyperbolic space.

$$H^{n'} = \exp_0^k(H^n)$$

$$\exp_0^k : \mathcal{T}_0\mathbb{L}_k^n \to \mathbb{L}_k^m$$

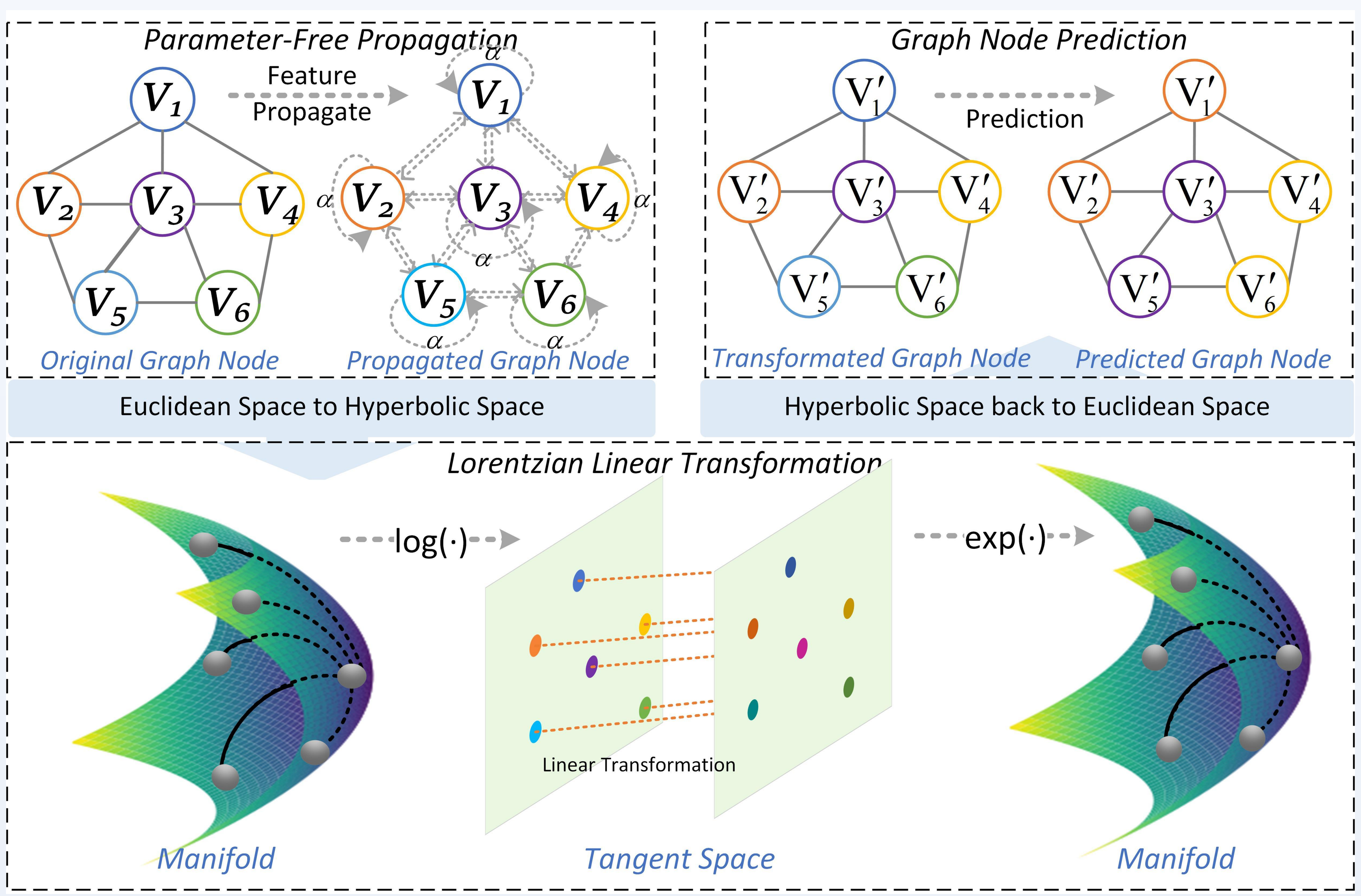
"k" is the curvature of hyperbolic space.

- Proposing a Lorenrzian linear transformation to capture the tree-like structure of data.

$$H^{n''} = M^{\otimes^k}(H^{n'})$$

$M^{\otimes^k}$ represents Lorenrzian linear transformation.

- Graph node prediction.



Experiments

Semi-supervised node classification.

(See the paper for a full description)

Space	Type	Method	Cora $\delta=11$	Citeseer $\delta=4.5$	PubMed $\delta=3.4$
Euclidean	Nonlinear	GCN(Kipf and Welling, 2017)	81.5	70.3	79
		GAT(Veličković et al., 2018)	83.0	72.5 ± 0.7	79.0 ± 0.3
		SCGNN(Liu et al., 2023)	**84.5 ± 0.3**	73.5 ± 0.5	80.8 ± 0.5
	Linear	SGC(Wu et al., 2019)	81.0 ± 0.0	71.9 ± 0.1	78.9 ± 0.0
		DGC(Wang et al., 2021b)	83.3 ± 0.0	73.3 ± 0.1	80.3 ± 0.1
		G²CN(Li et al., 2022)	82.7	73.8	80.4
Hyperbolic	Nonlinear	HGCN(Chami et al., 2019)	81.3 ± 0.6	70.9 ± 0.6	78.4 ± 0.4
		HAT(Zhang et al., 2021a)	83.1 ± 0.6	71.9 ± 0.6	78.6 ± 0.5
		HYBONET(Chen et al., 2022)	80.2 ± 1.3	-	78.0 ± 1.0
		HGCL(Liu et al., 2022)	82.3 ± 0.5	72.1 ± 0.6	79.14 ± 0.7
	Linear	**L²GC**(ours)	82.4 ± 0.0	**74.7 ± 0.0**	**81.3 ± 0.0**

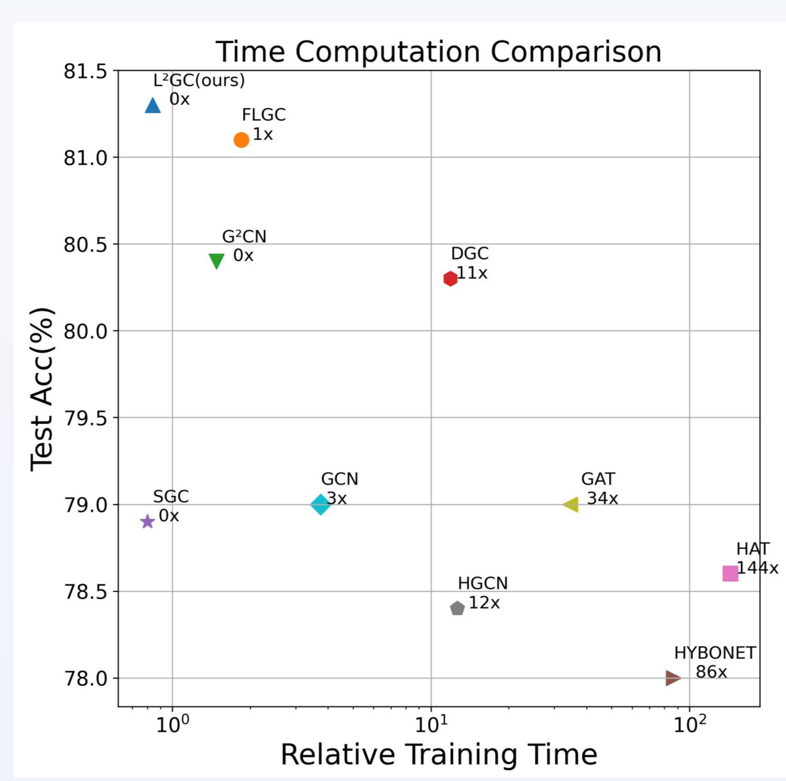
- Comparing with models in Euclidean space, our model outperforms the current SOTA linear model G²CN by 1.2% and 0.2% on Citeseer and PubMed.
- Comparing with models in hyperbolic space, our approach is 3.6% and 2.7% better than the SOTA model (HGCL) on Citeseer and PubMed datasets.

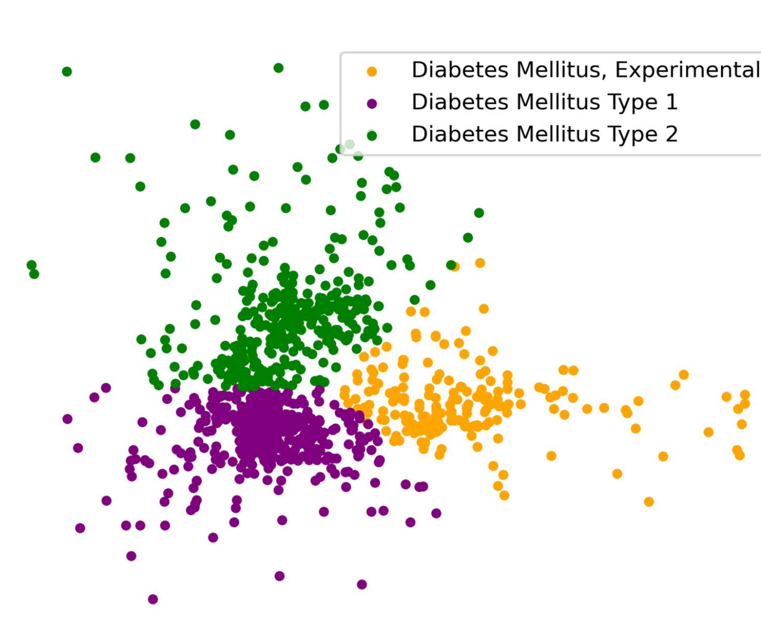
Fully supervised node classification.

Space	Method	Disease $\delta=0$	Airport $\delta=1$
$\mathbb{E}$	GCN(2017)	69.7 ± 0.4	81.4 ± 0.6
	GAT(2018)	70.4 ± 0.4	81.5 ± 0.3
	SGC(2019)	69.5 ± 0.2	80.6 ± 0.1
	SCGNN(2023)	85.3 ± 0.4	-
$\mathbb{H}$	HGCN(2019)	82.8 ± 0.8	90.6 ± 0.2
	HAT(2021a)	83.6 ± 0.9	-
	LGCN(2021b)	84.4 ± 0.8	90.9 ± 1.7
	HGCL(2022)	93.4 ± 0.8	92.3 ± 1.0
	HYBONET(2022)	**96.0 ± 1.0**	90.9 ± 1.4
	L²GC(ours)	94.4 ± 0.1	**94.0 ± 0.1**

- Our model achieves 3.4% and 1.8% over HYBONET and HGCL on accuracy.
- L²GC doesn't improve on HYBONET for the Disease dataset, but our model is faster and more stable.

Efficiency analysis (Left) and Visualisation (Right)





- L²GC is trained faster one or two orders of magnitude than other nonlinear models.
- L²GC can achieve better node classifcation results.